Ph.D. Synopsis

Title: Generalized Weight Distribution of Codes from Higher Dimensional Projective Varieties over Finite Fields

Abstract of Proposed Work Plan / Problem:

1. **Introduction:**Study of Linear Codes associated to higher dimensional varieties have been emerged as one of the interesting fields of research in mathematics. This subject brings together the fields such as algebraic geometry over finite fields, coding theory, and cryptography. I am interested to study linear codes defined from one of such higher dimensional algebraic varieties named as determinantal varieties which are described as follows: For a fixed prime power q, positive integers t, l, m & a $l \times m$ consider a matrix $X = (X_{ij})$ whose entries are independent indeterminates over the finite field $\mathbb{F}_q$.Let $\mathbb{F}_q[X]$denote the polynomial ring in lm variables X_{ij} $(1 \leq i \leq l, 1 \leq j \leq m)$ with coefficients in $\mathbb{F}_q$.A$t \times t$ minor of $X$is the determinant of a $t \times t$ submatrix of X, where t is a nonnegative integer $\leq \min\{l, m\}$. We shall always assume, without loss of generality, that$l \leq m$. Let $\mathbb{M}_{l\times m}(\mathbb{F}_q)$ the set of all $l \times m$ matrices with entries $\mathbb{F}_q$ and in this case we may simply write $\mathbb{M}_{l\times m}$for$\mathbb{M}_{l\times m}(\mathbb{F}_q)$. Note that $\mathbb{M}_{l\times m}$ can be viewed as an affine space$\mathbb{A}^{lm}$ over $\mathbb{F}_q$ of dimensionlm. For$0 \leq t \leq l$, the corresponding classical determinantal variety (over $\mathbb{F}_q$) is denoted by $\mathcal{D}_t$ and defined as the affine algebraic variety in $\mathbb{A}^{lm}$ given by the vanishing of all $(t+1) \times (t+1)$ minors of X; in other words

$$\mathcal{D}_t = \{M \in \mathbb{M}_{l\times m}(\mathbb{F}_q) : rank(M) \leq t\}.$$

The affine variety $\mathcal{D}_t$ is in fact, a cone; in other words, the vanishing ideal I_{t+1} (which is precisely the ideal of $\mathbb{F}_q[X]$ generated by all $(t+1) \times (t+1)$ minors of X) is a homogeneous ideal. Also it is a classical (and nontrivial) fact that I_{t+1} is a prime ideal. Thus $\mathcal{D}_t$ can also be viewed as a projective algebraic variety in $\mathbb{P}^{lm-1}$, and viewed this way, we will denote it by $\widehat{\mathcal{D}}_t$. We remark that the dimension of $\widehat{\mathcal{D}}_t$is $t(l+m-t)-1$. The determinantal code $\hat{C}_{det}(t; l, m)$ is the linear code corresponding to the projective system$\widehat{\mathcal{D}}_t \hookrightarrow \mathbb{P}^{lm-1}(\mathbb{F}_q) = \mathbb{P}(\mathbb{M}_{l\times m})$. This code is obtained by considering the image $C_{det}(t; l, m)$ of the evaluation map $Ev : \mathbb{F}_q[X]_1 \to \mathbb{F}_q^n$ defined by $Ev(f) = c_f := (f(M_1), \ldots, f(M_n))$, where $\mathbb{F}_q[X]_1$ denotes the space of homogeneous polynomials in $\mathbb{F}_q[X]$ of degree 1 together

with the zero polynomial, and $M_1, \ldots\ldots, M_n$ is an ordering of $\mathcal{D}_t$. The details are given in [1]. In this paper, the authors have determined the parameters length, dimension, weight distribution, spectrum of these codes defined from 2×2 minors. But, the problem of weight distribution and spectrum is open in general. I would like to pursue my research related to one of these problems.

2. **Review of literature and developments in the subject (Previous work done in the relevant area):** In [1], Linear Codes associated to Determinantal Varieties are studied by Beelen, Ghorpade, and Hasan. They consider a class of linear codes associated to projective algebraic varieties defined by the vanishing of minors of a fixed size of a generic matrix. It is seen that the resulting code has only a small number of distinct weights. The case of varieties defined by the vanishing of 2×2 minors is considered and obtain the complete weight distribution. Moreover, several generalized Hamming weights are determined explicitly and it is shown that the first few of them coincide with the distinct nonzero weights. In this view, I would like to explore the literature given below:
    i) S. Hasan, *Primitive Recursive Vector Sequences, Polynomial Systems And Determinantal Codes Over Finite Fields*, Ph. D. Thesis, IIT Bombay (2009).
    ii) P. Beelen, S.R. Ghorpade, S. Hasan, *Linear Codes Associated to Determinantal Varieties*, Discrete Mathematics, **338** (2015), 1493-1500.

3. **Objectives of Research / Proposed Hypothesis:** I am primarily interested to study the determinantal codes and would like to work on the problem of determining all the higher weights of detreminantal codes $\hat{C}_{det}(1; l, m)$ & $\hat{C}_{det}(t; l, m)$ for $1 \leq t \leq l$.
4. **Methodology to be adopted:** I will adopt the following methodology to pursue the studies for my doctoral work:
    Stage 1: Completion of Course Work as per the university rules.
    Stage 2: Literature Survey: I need to refer the following text books and papres in the area of my research:
    (i) K. Hoffman, R. Kunze, *Linear Algebra (Second Edition)*, Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1971.
    (ii) D. Dummit, R. Foote, *Abstract Algebra*, Wiley Publication (2004)

(iii) M. Artin, *Algebra*, Prentice-Hall, Upper Saddle River, New Jersey 07458 (1991)

(iv) R. Lidle, H. Niederreiter, *Finite Fields and their Applications,* CUP, Cambridge (1986)

(v) D. Cox, J. Little, D. O'shea, *Ideals, Varieties and Algorithms,* Springer Publication, New York (2007)

(vi) Joe Harris, *Algebraic Geometry*, Springer – Verlag Publication, New York

(vii) S. Hasan, *Primitive Recursive Vector Sequences, Polynomial Systems And Determinantal Codes Over Finite Fields*, Ph. D. Thesis, IIT Bombay (2009)

(viii) P. Beelen, S.R. Ghorpade, S. Hasan, *Linear Codes Associated to Determinantal Varieties*, Discrete Mathematics, **338** (2015), 1493-1500

Stage 4: In this stage I may be in a position to submit my research work to the committee at the research center of SRTMU, Nanded for the review and comments.

5. **Importance of study / Society Applications:** The study of error correcting codes associated with the algebraic curves is a popular area of research at the international & national level. Because, the linear codes from algebraic curves are theoretically strong in terms of their error correction & detection capabilities. We can also study the problems in Algebraic Geometry due to the correspondence between Coding Theory & Algebraic Geometry.

6. **Proposed Work Plan / Formulation & Structure of study:**
   a. **Ist year :** Course Work as per the university rules
   b. **IIndyear:** Literature survey related to problems (The details are given in item 5 of this report)
   c. **IIIrdyear:** Submission of research work for the review

7. **References:**

[1] P. Beelen, S.R. Ghorpade, S. Hasan, Linear Codes Associated to Determinantal Varieties, Discrete Mathematics, 338 (2015), 1493-1500, Linear codes associated to determinantal varieties, Vol. 338 (2015), 1493-1500.

[2] S. Hasan, *Primitive Recursive Vector Sequences, Polynomial Systems and Determinantal Codes over Finite Fields*, Ph. D. Thesis, IIT Bombay (2009).

[3] D. Yu. Nogin*, Codes associated to Grassmanians*, Arithmetic, Geometry and CodingTheory R. Pellikan, M. Perret, S.G. Vladut. Eds., 145-154, Walter de Gruyter, Berlin/ NewYork (1996).

[4] J.W.P. Hirschefeld and J.A. Thas, General Galois Geometries, Oxford Mathematical Monographs.

[5] C.T. Rayan, K.M. Rayan, *The minimum weight of Grassmann Codes C(k,n),* Discrete Appl. Math. 28, 149-156, 1990.

Ms. Sharmishtha S. Deshmukh
Research Scholar

Dr. Arunkumar R. Patil
Research Guide